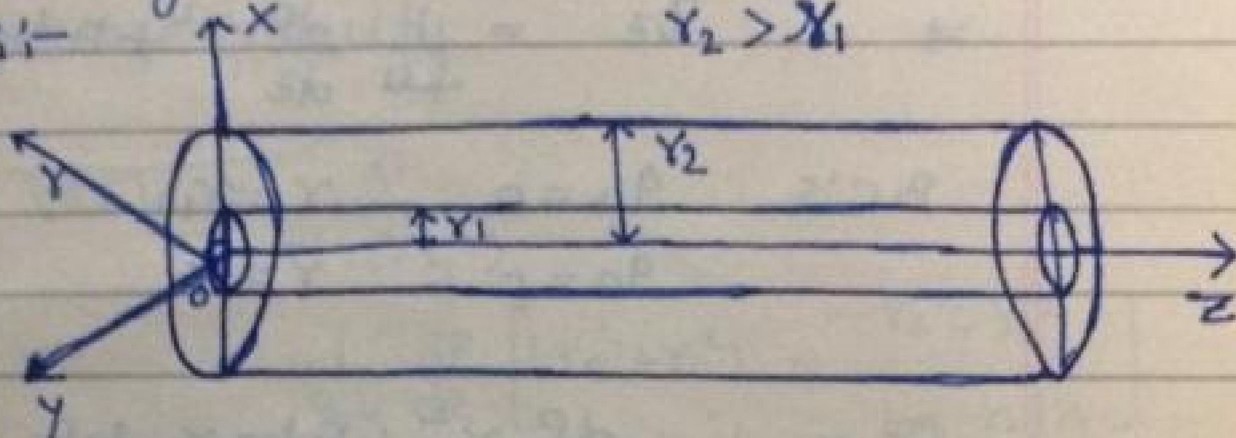
Shear stress:-

$$(\tau_{rz})_{r=r_0} = -\mu \frac{dq_z}{dr}$$

$$= -\frac{1}{2} r_0 \frac{dP}{dz}$$

Case I:- Steady flow between co-axial circular Pipes:-

$r_2 > r_1$



Consider two co-axial circular cylinder of radius r_1 And r_2 $(r_2 > r_1)$ through which laminar steady flow without body forces on in compressible fluid takes place along the axial directions.

the boundaries of both the pipese arealong at rest. the no slip conditions at their walls become

Equation of motion

$$0 = -\frac{\partial P}{\partial r} \Rightarrow P \text{ is Independent of } r$$

$$0 = -\frac{1}{r}\frac{\partial P}{\partial \theta} \Rightarrow P \text{ is Independent of } \theta$$

$$0 = \frac{-1}{4\mu}\frac{dP}{dz} \cancel{\quad} + \mu\left(\frac{d^2 q_z}{dr^2} + \frac{1}{r}\frac{dq_z}{dr}\right)$$

the Pressure P is a function of z only.

i-e $P = P(z)$

Average velocity $\boxed{V = U\left(\frac{1}{2} + \frac{P}{6}\right)}$

the Volumetric flow rate Q/unit width

$$Q = VY_0 = \left(\frac{1}{2} + \frac{P}{6}\right) UY_0$$

For maximum or minimum velocity

$\frac{du}{dy} = 0$ we have

$$\frac{du}{dy} = \frac{U}{Y_0} + \frac{U}{Y_0} P\left(1 - \frac{2Y}{Y_0}\right) = 0$$

$$= U\left[\frac{1}{Y_0} + P\left(\frac{1}{Y_0} - \frac{2Y}{Y_0^2}\right)\right] = 0$$

$$\Rightarrow \frac{Y}{Y_0} = \frac{1}{2}\left(1 + \frac{1}{P}\right)$$

the maximum velocity $P = 1$

$$\frac{Y}{Y_0} = \frac{1}{2}(1+1)$$

$$\frac{Y}{Y_0} = 1$$

$$Y = Y_0$$

For minimum velocity $P = -1$

$$\frac{Y}{Y_0} = \frac{1}{2}\left[1 - \frac{1}{1}\right]$$

$$\frac{Y}{Y_0} = 0$$

~~It follows~~ ⇒ the velocity gradient at ~~the~~ fixed plate is zero. it becomes negative for any value $P < -1$

And $0 = \frac{1}{8\mu} \frac{dp}{dx} y_0^2 - \frac{1}{2} A y_0 + B$ —— (B)

From A And B

$$B = \frac{-1}{8\mu} \frac{dp}{dx} y_0^2$$

$$A = 0$$

Substituting A And B Value in the

$$u = \frac{1}{2\mu} \frac{dp}{dx} y^2 + Ay + B$$

$$u = \frac{1}{2\mu} \frac{dp}{dx} y^2 - \frac{1}{8\mu} \frac{dp}{dx} y_0^2$$

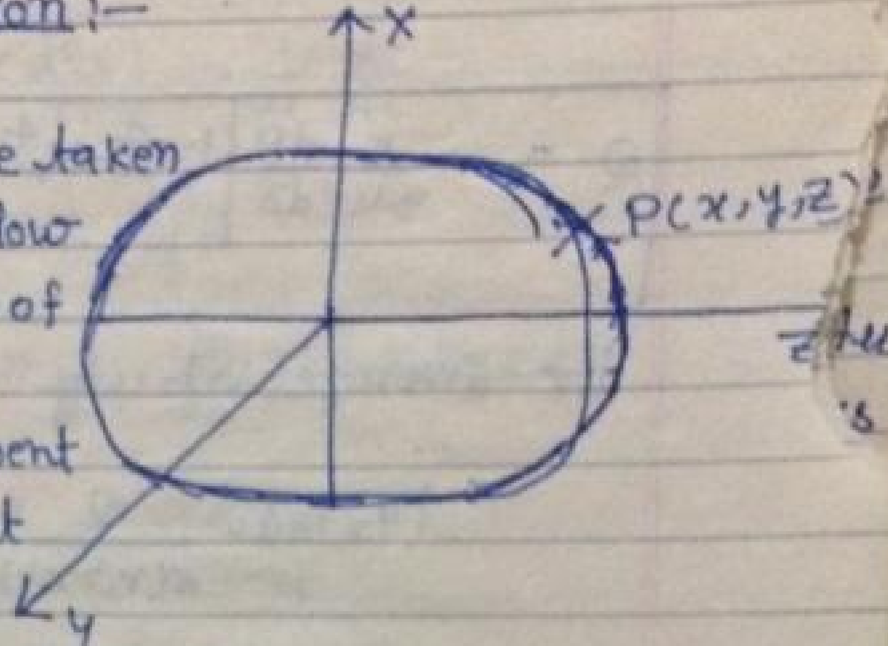
$$\boxed{u = -\frac{y_0^2}{8\mu} \frac{dp}{dx} \left(1 - \frac{4y^2}{y_0^2}\right)}$$

it follows that the fluid flows in the direction on the negative pressure gradient and that the velocity profile of the fully developed laminar flow between two parallel plates.

Ques:- State And Proved the Poiseuille fl between two parallel plates.

=x=

Case II Steady flow in Pipes of elliptic Cross - section :-



Let z-axis be taken direction of flow Along the axis of the pipe. w is velocity component velocity componet w is indepen dent z

the Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Velo. comp.

$$u = 0, \; v = 0, \; w = w(x, y) \quad \text{——} ①$$

$$\frac{dw}{dz} = 0 \Rightarrow w \text{ is independent } z$$

the NSE

$$0 = -\frac{\partial p}{\partial x} \quad \text{——} ②$$

$$0 = \frac{-\partial p}{\partial y} \quad \text{——} ③$$

$$0 = \frac{-\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \text{——} ④$$

Differentiating the equation ④ with rigard to z, we have

$$\frac{d^2 p}{dz^2} = 0$$

$$\Rightarrow \frac{dp}{dz} = \text{const.} = -c$$

the Pressure gradient

$$\frac{dP}{dz} = \frac{P_2 - P_1}{L}$$

$$q_z = -\frac{r_0^2}{4\mu}\frac{P_2 - P_1}{L}\left(1 - \frac{r^2}{r_0^2}\right)$$

Where P_1 And P_2 are Pressure

Maximum velocity

$$(q_z)_{max} = -\frac{r_0^2}{4\mu}\frac{dP}{dz}, \quad \frac{dP}{dz} < 0$$

$$\frac{q_z}{(q_z)_{max}} = 1 - \frac{r^2}{r_0^2}$$

this is Known as Hagen- Paiseuille flow

Average velocity

$$(q_z)_{av} = \frac{1}{\pi r_0^2}\int_0^{2\pi}\int_0^{r_0} q_z \, r \, dr \, d\theta$$

$$\Rightarrow (q_z)_{av} = \frac{1}{\pi r_0^2}\int_0^{2\pi}\int_0^{r_0} -\frac{r_0^2}{4\mu}\frac{dP}{dz}\left(1 - \frac{r^2}{r_0^2}\right) r \, dr \, d\theta$$

$$\Rightarrow (q_z)_{av} = \frac{-1}{2\mu}\frac{dP}{dz}\left(\frac{1}{2}r_0^2 - \frac{1}{4}r_0^2\right)$$

$$= \frac{1}{2}\left(-\frac{r_0^2}{4\mu}\frac{dP}{dz}\right)$$

$$= \frac{1}{2}(q_z)_{max}$$

the volumetric flow Q

$$Q = \frac{1}{2}(q_z)_{max}\,\pi r_0^2$$

$$\boxed{Q = -\frac{\pi r_0^4}{8\mu}\frac{dP}{dz}}$$